1st Hr

4.7 Inverse Trigonometric Functions

Recall that in order for a function to have an inverse function, it must be one-to-one (it must pass the Horizontal Line Test). Therefore, in order for the function, $y = \sin x$ to have an inverse, we must restrict the domain.

When restricting the domain to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$, the following properties hold.

★ inverse graphs are in textbook Pg. 378-380

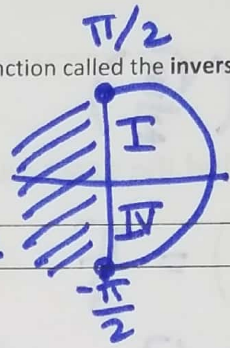
On the interval $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$:

1. $y = \sin x$ is increasing.
2. The range of $y = \sin x$ is [-1, 1].
3. $y = \sin x$ is one-to-one.

So, restricting the domain of $y = \sin x$ to $-\frac{\pi}{2} \le x \le \frac{\pi}{2}$ gives a unique function called the **inverse sine function** denoted by

$y = \arcsin x$ or $y = \sin^{-1} x$

find θ between $-\pi/2$ to $\pi/2$



Definition of Inverse Sine Function

The **inverse sine function** is defined by

$y = \arcsin x$ if and only if $\sin y = x$

where $-1 \le x \le 1$ and $-\frac{\pi}{2} \le y \le \frac{\pi}{2}$. The domain of $y = \arcsin x$ is [-1, 1], and the range is $\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$.

The notation $\sin^{-1} x$ is consistent with the inverse function notation $f^{-1}(x)$.

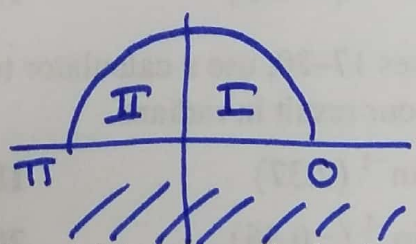
Note that $\sin^{-1} x$ means the inverse sine function and does not mean $\frac{1}{\sin x}$.

The $\arcsin x$ notation is read as "the arcsine of x" and comes from the association of a central angle with its intercepted *arc length* on a unit circle. In other words, $\arcsin x$ means the angle (or arc) whose sine is x. **Both notations, $\arcsin x$ and $\sin^{-1} x$, mean the same thing and are commonly used in mathematics to find an angle.**

When restricting the domain to $0 \le x \le \pi$, the following properties hold.

On the interval $[0, \pi]$:

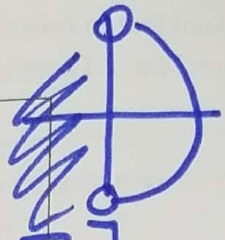
4. $y = \cos x$ is decreasing.
5. The range of $y = \cos x$ is [-1, 1].
6. $y = \cos x$ is one-to-one.



So, restricting the domain of $y = \cos x$ to $0 \le x \le \pi$ gives a unique function called the **inverse cosine function** denoted by

$y = \arccos x$ or $y = \cos^{-1} x$

Similarly, you can define an **inverse tangent function** by restricting the domain of $y = \tan x$ to the interval $\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$.



Definitions of the Inverse Trigonometric Functions

Function	Domain	Range	
$y = \arcsin x$ if and only if $\sin y = x$	$-1 \le x \le 1$	$-\frac{\pi}{2} \le y \le \frac{\pi}{2}$	$\left[-\frac{\pi}{2}, \frac{\pi}{2}\right]$
$y = \arccos x$ if and only if $\cos y = x$	$-1 \le x \le 1$	$0 \le y \le \pi$	$[0, \pi]$
$y = \arctan x$ if and only if $\tan y = x$	$-\infty < x < \infty$	$-\frac{\pi}{2} < y < \frac{\pi}{2}$	$\left(-\frac{\pi}{2}, \frac{\pi}{2}\right)$

$\arcsin\left(\frac{\sqrt{3}}{2}\right)$

looking for θ in the correct restrictions that has a certain value.

In Exercises 1–12, find the exact value.

1. $\sin^{-1}\left(\frac{\sqrt{3}}{2}\right) = \frac{\pi}{3}$
2. $\sin^{-1}\left(-\frac{1}{2}\right)$
3. $\tan^{-1} 0 = 0$
4. $\cos^{-1} 1$
5. $\cos^{-1}\left(\frac{1}{2}\right) = \frac{\pi}{3}$
6. $\tan^{-1} 1$
7. $\tan^{-1}(-1) = -\frac{\pi}{4}$
8. $\cos^{-1}\left(-\frac{\sqrt{3}}{2}\right)$
9. $\sin^{-1}\left(-\frac{1}{\sqrt{2}}\right) = -\frac{\pi}{4}$ $\quad \sin^{-1}\left(-\frac{\sqrt{2}}{2}\right)$
10. $\tan^{-1}(-\sqrt{3})$
11. $\cos^{-1} 0 = \frac{\pi}{2}$
12. $\sin^{-1} 1$

HW #2-12 even

In Exercises 13–16, use a calculator to find the approximate value. Express your answer in degrees.

13. $\sin^{-1}(0.362)$
14. $\arcsin 0.67$
15. $\tan^{-1}(-12.5)$
16. $\cos^{-1}(-0.23)$

In Exercises 17–20, use a calculator to find the approximate value. Express your result in radians.

17. $\tan^{-1}(2.37)$
18. $\tan^{-1}(22.8)$
19. $\sin^{-1}(-0.46)$
20. $\cos^{-1}(-0.853)$